RAN-2203000205023002

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T. Y. B. Sc. (Mathematics) (Sem. - V) Examination March - 2023

MTH - 502 : Linear Algebra - I

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1) નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **T. Y. B. Sc. (Mathematics) (Sem. - V)**

Name of the Subject :

☛ **MTH - 502 : Linear Algebra - I**

Subject Code No.: **2203000205023002**

Seat No.:

Student's Signature

(2) All questions are compulsory.
(3) Figures to the right indicate marks of the questions.
(4) Follow usual notations.

Q. 1. Answer the following : (Any Five) **(10)**

(1) Is $x * y = (x + y) - (x \cdot y) \in N$? $\forall\ x, y \in N$. Justify your answer.

(2) In a vector space R^+, for every $u, v \in R^+$ if $u + v = u \cdot v$ then find Identity and Inverse element with respect to the operation addition.

(3) Is $M = \{(x_1, x_2) \in V_2 / x_2 \neq 0)\}$ be a subspace of V_2? Justify your answer.

(4) Find the span of X-axis and the plane $x + y = 0$ in vector space V_3.

(5) For what values of values of

$\alpha_i 1v_1 + 2v_2 + 3v_3 + ----- + \alpha_i v_i + ----- + nv_n$; $n \in N$ be a Non trivial linear combination?

(6) Let $U = xy$ – plane and $W = xz$ – plane. Is $V_3 = U \oplus W$? Justify your answer.

(7) Is a basis can never include the zero vector? Justify your answer.

(8) Let U and W be two distinct $(n - 1)$ dimensional subspace of an n –Dimensional vector space V. Then find the dimension of $\dim(U \cap W)$.

Q. 2. **Answer the following : (Any two)** **(10)**

(1) Prove that the set $\{(x_1, x_2, x_3, x_4, x_5) \in V_5 / x_i \in R, \forall\, i = 1 \text{ to } 5\}$ be an abelian group with respect to coordinate wise addition.

(2) Prove that the set $\{\alpha_1 u_1 + \alpha_1 u_2 + --- + \alpha_n u_n / \alpha_i \in R, u_i \in V, \forall\, i = 1 \text{ to } n\}$ is a subspace of vector space V.

(3) Let U and W be two subspace of vector space V then prove that $U + W = U$ if and only if $W \subset U$. State the necessary condition for $U \cup W$ be a subspace of V.

Q. 3. **Answer the following : (Any two)** **(10)**

(1) Let S be a non empty subset of a vector space V then prove that the span [S] is a smallest subspace of V containing S.

(2) Determine which of the following vector belongs to the span of a set S = {(1,2,1), (1,1,–1), (4,5,–2)} (*a*) $\left(\frac{-1}{3}, \frac{-1}{3}, \frac{1}{3}\right)$ (*b*) (1,–3,5)

(3) If U and W are two subspaces of a vector space V then prove that $U + W$ is a subspace of V with condition that $U + W = [U \cup W]$.

Q. 4. **Answer the following : (Any two)** **(10)**

(1) In a vector space V, prove that

(a) If $v_1 = v_2 + v_3$ then the set $\{v_1, v_2, v_3)$ is L.D.

(b) If the set $\{v_1, v_2, ..., v_n\}$ is L, I and $v \in [v_1, v_2, ..., v_n]$ in a vector space V then the set $\{v, v_1, v_2, ..., v_n\}$ is L.D.

(2) In a vector space V, suppose $\{v_1, v_2, ..., v_n\}$ is an ordered set of vectors with $v_1 \neq 0$. If the set $\{v_1, v_2, ..., v_n\}$ is L.D then prove that one of the vectors from $v_2, v_3, ..., , v_n$, say $v_k \in [v_1, v_2, ..., v_{k-1}]$, $2 \leq k \leq n$.

(3) If the set $S_4 = \{(1,1,0), (0,1,1), (1,1,-1), (1,1,1)\}$ is L.D then find largest L.I subset S_3 of S_4 with condition $[S_3] = [S_4]$.

Q. 5. Answer the following : (Any two) **(10)**

(1) Define : Dimension of a vector space V. If a vector space V has a basis B_1 containing four elements then prove that every other basis B_2 has also four elements.

(2) (a) In a vector space V, the set $[B] = [v_1, v_2, ..., v_n] = V$. Then prove that the expression $v = \alpha_1 v_1 + \alpha_2 v_2 + ... + \alpha_n v_n$ is unique for every $v \in V$ then B is L.I.

(b) Is the set $\{(1,-1,-1), (3,3,3), (4,2,2)\}$ a basis of a vector space V_3? Justify.

(3) Find the general form of a co-ordinate vector of a vector (x_1, x_2, x_3) relative to a basis $\{(1,1,1), (1,-1,1), (0,1,1)\}$.
